3D quench simulation of superconducting magnets with adaptive time-stepping and mesh-generation

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Abstract — This paper presents a three-dimensional (3D) numerical method to simulate the quench behavior of superconducting magnets. To reduce the computation complexity, an algorithm is developed with adaptive time-stepping and mesh-generation according to the rise rate of temperature at the hot spot and the frontier boundary of quench propagation. 3D quench simulations of an example magnet are performed to illustrate the proposed method.

I. INTRODUCTION

Prediction of the behavior of a superconducting magnet during quench, that is, a transition from the superconducting state to the normal state, is important because this phenomenon may cause destruction of the magnet [1]. Various approaches have been proposed to simulate this electromagnetic-thermal-circuit coupled process, including semi-empirical methods [1, 2] for fast and approximate simulations and numerical codes based on the finite element method (FEM) [3, 4] for accurate calculations

To reduce the computation complexity of the 3D numerical simulation of the quench behavior, an adaptive time-stepping and mesh-generation method is developed in this paper. The detailed descriptions of the method are presented, and the quench simulations of an example magnet are demonstrated.

II. NUMERICAL MODEL

Under the assumptions that the magnet winding is made of a uniform but anisotropic material and it is in the adiabatic condition, the governing equations of the quench model of the magnet are given as follows.

1) Heat diffusion equation:

$$\rho C(T)\frac{\partial T}{\partial t} = Q_i(B,T,I)+\frac{1}{r}\frac{\partial}{\partial r}(rk_r(T)\frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial}{\partial \theta}(k_\theta(T)\frac{\partial T}{\partial \theta})+\frac{\partial}{\partial z}(k_z(T)\frac{\partial T}{\partial z}) \quad (1)$$

where t, T, r, z, θ are time, temperature, and the radial, axial and circumferential coordinates respectively; k_r, k_z, k_θ are the thermal conductivities; ρC is the heat capacity; and Q_i is the internal Ohmic heating power that is produced in the quenched zone of the winding. Q_i is a function of magnetic flux density, temperature and current.

2) Electrical circuit equations:

$$L\frac{dI}{dt}+IR(t)=0 \quad (2)$$

$$R(t)=\frac{1}{A_m}\int_0^l \rho(B,T)dl \quad (3)$$

where $L, I, R(t)$ is the inductance, current, and time-dependent normal zone resistance of the winding; A_m is the cross-section area of the conductor matrix; l is the total length of the normal zone conductors; and $\rho(B,T)$ is the electrical resistivity of the conductor matrix, which is a function of both temperature and magnetic flux density.

3) Magnetic field equation:

According to the geometry of the magnet winding, the field distribution function α at any location in the winding, produced by a unit current, is calculated in advance. α is used to reconstruct the magnetic field that varies with the current decay and is given by,

$$B(r,z,\theta,t)=\alpha(r,z,\theta)*I(t) \quad (4)$$

FEM is used to solve (1), while a fourth-order Runge-Kutta algorithm is used to solve (2).

III. TIME-STEPPING & MESH-GENERATION ADAPTION

A. Time-stepping adaption

The quench process of superconducting magnets can be divided into two distinct stages: one before the whole magnet quenching and the other after that.

In the first stage, because of the propagation of the normal zone boundary, very fine time steps have to be applied to detect the newly-quenched elements at the very first time. The time steps are set to be a constant value Δt_0 during this period.

In the second stage during which the whole magnet has quenched, with the decay of the current and the increase of the heat capacity of the winding, the temperature rise rate of the winding becomes smaller and smaller. An adaptive time-stepping algorithm is then used as described as follows,

1) calculate the rise rate of the hot spot temperature for the previous step $T_h^{'k-1}$as,

$$T_h^{'k-1}=\frac{\Delta t_{k-2}}{\Delta t_{k-1}+\Delta t_{k-2}}\left(\frac{T_h^k-T_h^{k-1}}{\Delta t_{k-1}}\right) + \frac{\Delta t_{k-1}}{\Delta t_{k-1}+\Delta t_{k-2}}\left(\frac{T_h^{k-1}-T_h^{k-2}}{\Delta t_{k-2}}\right) \quad (5)$$

2) the rise rate of the hot spot temperature for the current step $T_h^{'k}$ can be obtained using the trapezoid rule as,

$$T_h^{'k}=\frac{2}{\Delta t_{k-1}}(T_h^k-T_h^{k-1})-T_h^{'k-1} \quad (6)$$

3) then the step size for the next step is given by,

$$\Delta t_{k+1}=\gamma \Delta t_0/T_h^{'k} \quad (7)$$

where γ is an accelerating factor that determines the accuracy and the efficiency of the algorithm.

B. *Mesh-generation adaption*

A quench generally originates from a local point and then spreads in the winding. Before the whole winding becomes quenched, the superconducting region beyond the normal zone boundary remains. Its temperature keeps at the level of the cryogenic liquid until the boundary moves close. There is a gap between the boundary and the superconducting region where the temperature ranges from the critical values on the boundary to the liquid temperature. As the quench continues to spread, the frontier boundary of the quench propagation keeps moving outwards until it reaches the edges of the winding.

An adaptive mesh-generation algorithm is designed that "deletes" all the elements in the winding at the beginning of quench and "restores" them gradually by detecting the frontier boundary of the quench propagation. For simplification, a 2D case is illustrated though the algorithm can be generalized to 3D easily.

As shown in Fig. 1(a), the cross-section of the winding is divided into $N_r * N_z$ elements with the regular quadrilateral-shape. At the beginning, all the elements are "deleted". A quench is assumed to occur at the innermost central element. It spreads outwards as shown in Fig. 1(b) where the red elements are quenched and the yellow ones are the normal zone boundary. At the time step for the moment shown in Fig. 1(b), the "restored" elements obviously include the red and yellow ones. Besides, the elements in the frontier gap are also among the "restored". As the time goes to the next step, the frontier boundary of the quench propagation will cover more elements that should be "restored" predicatively in advance. The elements to be "restored" are predicted basically according to the speed of the quench propagation. Once all the elements of the winding are "restored", the mesh is fixed.

IV. RESULTS

To demonstrate the presented method, 3D quench simulations of an example magnet are performed that takes account of the quench propagation in the circumferential direction in addition. The magnet winding is made of NbTi wires of 1mm in thickness and 1.5mm in wideness. The winding has 50 layers and 120 turns per layer. Its inner radius, outer radius and axial length are 0.2m, 0.255m and 0.186m respectively. The operating current is 272A and the magnetic field at the center of the magnet is 4.2T. According to the critical current values at various magnetic fields, the current safety margin at the highest field in the winding is about 40%.

Due to the symmetry of the winding, only a quarter of the magnet is modeled for simulations. The winding model is divided into 50, 60 and 15 segments in the radial, axial and circumferential directions respectively. The optimized finest time-step size is 2ms.

A quench is started at the innermost central point of the winding. Simulations of the quench process are performed firstly with the optimized finest time-step and the whole elements of the winding model, and then with both time-step

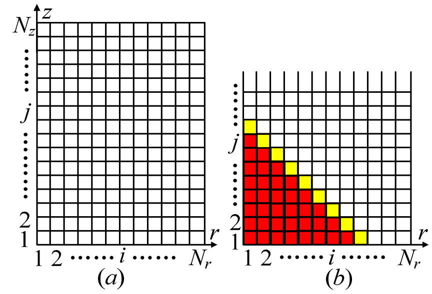

Fig. 1. Schematic diagram of the adaptive mesh-generation.

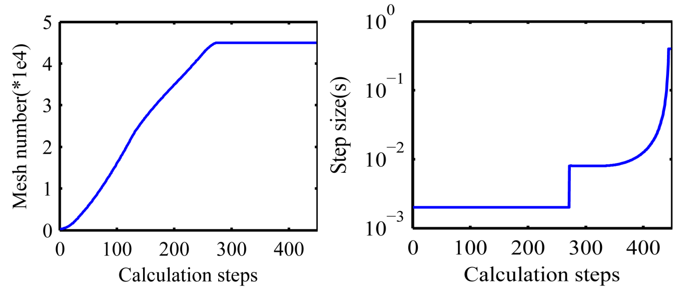

Fig. 2. Element number (left) and time-step size (right) vs. calculation steps.

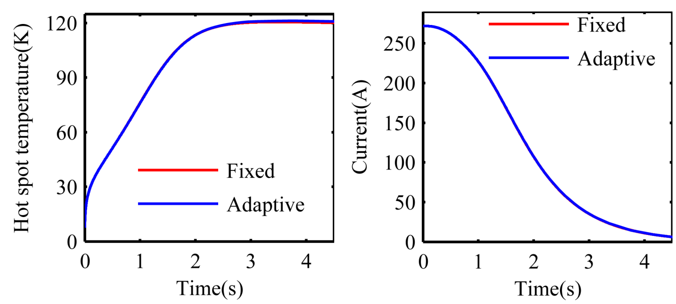

Fig. 3. Comparisons of the hot spot temperature (left) and the current (right).

and mesh adapted with the proposed method. Under a 3.0GHz PC, the simulated results of both cases are obtained and compared. The total simulation time with fixed time-step and mesh is 74hr (2500 calculation steps), while the time with the adaption is only 9hr (448 calculation steps). Fig. 2 shows the adapted results of the element number and time-step size with the calculation steps. It can be seen that the mesh-generation adaptive algorithm works in the first stage of the quench while the time-step adaption works in the second stage. Fig. 3 shows the comparisons of the hot spot temperature and the current between the two methods. Compared with the fixed time-step and mesh case, the computation time is dramatically shortened keeping the same computation accuracy by the proposed adaptive method.

V. REFERENCES

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